Three-dimensional topology optimisation in architectural and structural design of concrete structures

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1. Introduction

The research presented in this paper is part of a larger research effort into the development of doubly curved structures cast in concrete. This paper concentrates on the work carried out on the subject of using topology optimisation in architectural design and its subsequent realisation in concrete.

The aim of the research project is to investigate whether topology optimisation based on existing software solutions can be used as an architectural design tool for concrete structures; and whether large scale CNC-milling of polystyrene casting moulds is an applicable technique for the realisation of optimised shapes.

1.1 Assumptions

The following assumptions regarding the potential significance of implementing optimisation in the architectural design process serves as a background for the research:

Topology optimisation within architectural design is the use of topological optimisation techniques within the early processes of a building design. This as a tool to convey not only structural coherence, but also aesthetic qualities specific to the morphology of optimised shapes. Topology optimisation offers considerable potential within architectural design as a driver of design innovation and the convergence of the architectural and engineering disciplines.
Topology optimisation (figure 1) allows for the evolution of ‘structural shape’, i.e. shape that simultaneously manifests a structural optimum and an aesthetic assertion. As aesthetic values cannot reasonably be validated through numerical evaluation, only structural criteria can be directly utilized as an objective for an algorithmic optimisation process. But as the optimisation process itself is a linear result of the optimisation algorithm, aesthetic reflections can be indirectly imbedded within the calculation by evaluating the initial optimisation output, and then applying adjustments to the model.

![Figure 1: The topology optimisation process. From an initial set up of design- and non-design space, the optimisation software computes an optimal distribution of material in relation to design criteria.](image)

### 2. Optimisation examples

Existing optimisation software is developed in preparation for the automotive, aeronautic and naval industries, focussing on the use of isotropic materials with homogenous compressive- and tensile strength properties.

The optimisation tools are not specifically developed to meet the design of building structures with use of anisotropic or composite materials such as wood or reinforced concrete.

A research objective was to investigate, whether it is possible to use existing software for building related optimisation.

The research has focused on the application of topology optimisation to well known, simple, three dimensional cases based on pre-fabricated, post-tensioned concrete elements, as these are comprehensible to formulate and easily comparable with existing examples in shape as well as properties.
2.1 Method
The following method has been used for the optimization process. The concrete volume has been optimised only for the serviceability limit state, targeting minimum deformation energy and an un-cracked sections under long term loading.

Within the design process the post-tensioning has be adjusted until the demand for an un-cracked structure has been met. Further, a constraint of 1/400 of the largest span for the maximum allowable displacement has been used.

The topology optimisation was carried out using the commercial software OptiStruct, by the American software developer Altair Engineering, Inc.

The problems has been modeled using two types of spaces or material if one likes:

The design space is a three-dimensional volume, where the topology can be varied, and thus material removed. The non-design space is a volume with a constant material density that is included in the calculation, but not subjected to optimisation. In this way, the predefinition of necessary material is possible in areas where removal of material is undesirable, such as the deck of a bridge.

2.2. Example 1: Simply supported prestressed concrete slab
The first structure to be subjected to the topology optimization was a slim, simply supported concrete slab measuring 12 x 1.2 x 0.4 m (l * w *h) (figure 4). The slab was subjected to a single load case, in which a combined dead load (9.6 kN/m2) and live load (5.0 kN/m2) distributed evenly on the surface of the slab (Figure 4).

Post-tensioning was applied as single point forces, in all load cases, acting on the ends of the slab, with a magnitude equal to the energy derived from post-tensioning cables (1100 kN in each end).

![Figure 4: Optimisation setup of example 1.](image-url)
Material properties were in this initial case defined using a hypothetical concrete with identical strength in both compression and tension (60 MPa). The slab was modelled with a 50 mm non-design space at the top and at the supports. The definition of the described non-design space ensures that the optimisation would not affect the usability of the top surface, or the compatibility of the abutments with existing building systems.

The resulting form was characterised by a branching structure at the beam ends that resists the pressure from the post-tensioning forces. The branches then converge into beam-column that again diverges to the sides in the middle 1/3 of the slab (figure 5).

![Figure 5: CNC-milled model of optimised slab.](image)

The morphology of the optimized design is strongly organic, and characterized by sinuous curves that meander in contracting and diverging sequences. The emerged shape catches the light in multiple ways and creates a significant interplay between light and shadow in the shape. The assembling of the optimized elements into a loft structure creates a sculptural formation that allows the eye of the viewer to follow the trajectories of the forces within the physical form itself (figure 6).

![Figure 6: Visualisation of loft structure, formed by a series of optimised slabs.](image)
2.3. Example 2: Simply supported prestressed concrete beam

The second optimisation example was a simply supported prestressed concrete beam, measuring 12 x 0.3 x 1.2 m (l * w * h).

The problem was modeled with a permanent load distributed evenly on the top of the beam, comprising a dead load of 45 kN/m and a live load of 30 kN/m. Post-tensioning was applied at the bottom of the beam at a magnitude of 1500 kN and at the top of 700kN (figure 7).

Figure 7: Optimisation setup of example 2.

A 150 mm nondesign frame was established around the designspace to ensure usability of the optimised structure.

The initial optimization objective was to minimize deformation energy within a design constraint on the volume fraction of 40% of the total volume.

Within this model, we conducted a series of comparative studies of the impact of certain alterations in the configuration of the resulting topology, in order to test the difference in performance of the derived designs. Presented in the scheme below are the variables:

- Distribution of load
- Minimum size of basic elements
- Constraint on displacement

We tested the optimisation parameters on 2 examples (01 and 02), in which the first represented an even distribution of the load on the top of the beam, whereas the same load was applied in the centerpoint in example 2. Example 2 was subjected to an increased magnitude of post-tensioning (3000kN) in order to compensate for the stronger displacement produced by the centered load. In figure 8, 3 variations of each example are listed. The top variation contains no specific constraint on the optimisation process besides the volume fraction. The second line of variations is constrained with a minimum member
size that forces the topology to cluster into larger segments, redistributing material from sections with small formations into the larger. The third line of variations is constrained with a limit on the displacement of the center of the beam, set slightly above the results found in the first line of variations. In this way, the optimisation is forced to redistribute material in order to meet these added criteria, while keeping the initial constraints as well.

As is visible from the scheme, the initial topology contains a mixture of both large and thin segments, that in this case would be difficult to manufacture, as the forms are to be cast in concrete.

The introduction of a constraint on the minimum member size, creates a larger and more contiguous form that is easier to manufacture, yet at the cost of structural performance (increase of displacement).

The introduction of local constraints on displacement at the center (which structurally is not necessary as the initial model was within accepted limits of Danish codes), forces the topology to redistribute material with a slight decrease in local displacement at the constrained areas as a result. This generates a more complex geometry, with a finer network of interconnected segments. Though this might increase difficulties of production, the topological changes might as well contribute with a rise in aesthetic potential.
The difficulties of manufacturability can be compensated by adding a combination of both displacement constraint and minimum member size constraint. The beam rendered on figure 9 is the result of such combination. At the cost of an insignificant increase in displacement values (25.24 mm compared to initial 21.49), the design is enriched with an organic network of crossing diagonals and manufacturability is ensured due to the element size of the network. As such the resulting topology reflects the flow of forces: in the middle, where shear forces are low, the shear is resisted by a compression arch, leaving a void in the centre of the beam. Around the arch, where shear forces are increasing, the forces are counteracted by the truss-like structure of crossing diagonals (figure 9).

2.4. Example 3: Flat, pre-stressed concrete slab

The third optimisation example was a flat, pre-stressed concrete slab supported by 4 corner columns, measuring 9 x 6 x 0.4 m (l * w * h).

This example shows how practical and aesthetical concerns can be negotiated via a precise dialogue between evaluation of optimisation results and adjustments of optimisation setup.

In the initial setup, the slab was subjected to an evenly distributed load on the top, equal to a combined dead load (6.6 kN/m2) and live load (4.0 kN/m2).

Post-tensioning was evenly distributed on the sides (332 kN/m on the short side, 135.5 kN/m on the long side), and a 50 mm non-design space was established at the top. The optimisation objective was to minimize deformation energy with a design constraint of 30% of the total volume (figure 10).

The setup generated the design seen in figure 11, in which an oval void is created in the middle of the slab, interrupted by 4 constructional pins at the centre of each side that resists the pressure from the post-tensioning cables.
Though this design from a formal point of view has a strong and easily recognizable visual character that makes it an interesting object of design, it contains the obvious problem that because post-tensioning is evenly distributed on the sides, post-tensioning cables would need to run freely in the air through the void, inducing both fire risk and disturbing the aesthetics of the slab.

To encounter this, we reconfigured the model so that a 150 mm wide non-design frame was introduced at the sides, and all post-tensioning was concentrated in this frame. In this way, the changes in topology generated by the optimisation process would not affect the ducts for the post-tensioning cables.

The optimised result is shown in figure 12. Where now the practical issues regarding pre-stressing were solved, the graphical impact and character of the previous design was now reduced to a rather dull and indifferent curvature on the two sides of the slab, posing a significant reduction in the aesthetic quality of the design.
This illustrates the difficulty of using the optimized topologies as a form generator: though the shape is always structurally optimal (assuming setup is correct), the aesthetics of the shape may be far from that. To the architectural designer, reducing the aesthetics of a design during the design process is equally as unacceptable as reducing stability of the structure during the structural design process would be to the engineer. In order to avoid the two aspects from conflicting, possibilities of interpretation are needed to negotiate both the logic of structure and the aspects of beauty.

In the case of example 3, we chose to apply a second design constraint in order to force the topology to change, yet without changing the general optimisation conditions that were already defined. We chose to put a limitation on the displacement of the exact centre of the slab that was just slightly above the displacements tolerated by the codes.

The resulting change in topology is shown in figure 13. The two discrete curvatures of the earlier design are now connected via a crossing structure that resists the displacement of the centre. Where the crossing structure contains only minor improvement to the overall static performance of the structure, it poses a great improvement to the visual impact of the form. As opposed to the previous revised topology, the optimised form now has a clear and easily recognizable visual identity, and the proportions of the geometry is balanced, creating a subtle, sculptural appearance when assembled into a loft structure (figure 13).

Figure 13: Computer rendering of loft structure of second revised optimisation result
3. Production of optimised forms
The optimised forms shown in the above examples can be produced using large scale CNC-milling of polystyrene blocks that work as moulds for casting concrete. Included in the software used for this research is a variety of production constraints, such as to include e.g. 1- or 2-sided casting directions. A 1-sided casting direction constraint has been used in example 1 – 3, to prepare the form to a 1 sided casting in polystyrene moulds.

The high technology concrete workshop at the Danish Institute of Technology is equipped with a large scale CNC-milling robot (figure 14) that is utilized for this research. Parallel research on robot control, software translation of 3d-models and surface treatment of the casting moulds is now being conducted by our research partners.

Figure 14: Large Scale CNC-milling robot at the High Technology Concrete Workshop at the Danish Institute of Technology

The optimisation experiments conducted by the Aarhus School of Architecture will be concluded in an optimised, full-scale spatial structure that will be produced via the above mentioned facilities in a 1:1-prototype in the fall of 2009.
4. Conclusions

The preliminary investigations presented here show:

- That existing optimisation software commercially available within other industries can be utilized to model and calculate optimisation of building related structures.

- That topology optimisation is an applicable procedure in the development of new, sculptural structures of prestressed concrete.

- That topology optimisation contains possibility of a significant reduction in building material usage.

- That varying optimisation criteria can be utilized to push topological results in new directions – thus influencing the aesthetics of the appearance – without undermining the initial design constraint.

- That topology optimisation so allow for a direct and precise dialogue between aesthetic and structural considerations.

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This article is an excerpt from the research conducted at the Aarhus School of Architecture within the collaborate research project ‘Unikabeton – Industrial production of unique concrete elements via digital fabrication’. The partners participating in the project are: The Danish Institute of Technology; University of Southern Denmark; Aarhus School of Architecture; the companies Unicon A/S (concrete production); Spæncom A/S (production of concrete elements); MT Højgaard A/S (contractor); Paschal Denmark A/S (formwork); and Gibotech A/S (robotics). The project is funded by the Danish National Advanced Technology Foundation. The aim of the project is to investigate and develop industrialized techniques for fabrication of individualized concrete members and structures with complex geometry. The Aarhus School of Architecture contributes to the research by investigating topology optimisation in architectural design and the fabrication of optimised concrete structures.